Deviation Cycles in Non-Scheduled Worked Hours

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Abstract

This paper attempts to estimate the probabilities of economic slack in the labor market of Japan and examines how useful they are to understand the business cycle. We use Markov switching models to estimate them with a deviation-cycle data of non-scheduled worked hours. The main findings are as follows. First, a simple Markov switching model generates the smoothed probabilities that are closely related to the business cycle. Second, inclusion of lagged dependent variables as explanatory variables does not solve the model misspecification problem indicated by the conventional specification testing. Third, a Markov switching model statistically fits the data with frequency components higher than seasonality better than those without them. The models accepted by specification testing, however, do not generate the smoothed probabilities consistent with the business cycle.

Key words: economic slack, deviation cycles, Markov switching

JEL classification: C51, E32

1 Introduction

The reference dates of business cycle are not only a starting point of empirical research, but a great concern from the press and policymakers. Particularly, significant downturns in economic activity are a fundamental concern making the headlines. Recently, Romer and Romer (2019) proposed that the NBER (National Bureau of Economic Research, U.S.A.) should focus on a large and rapid rise in economic slack to date peaks and troughs of business cycles, rather than on a decline in economic activity as that the modern definition emphasizes. They pointed out that the dates played an important role in establishing the concept of a recession as a repeated and identifiable phenomenon.

Furthermore, they claimed that the focus on the economic slack lead to a narrower and more precise definition of a recession that is more firmly grounded in modern understanding of macroeconomic. They showed some supportive evidence for the United States and Japan in the modern low-growth era. They argued that it appeared not only better suited to identifying episodes of interest in settings where trend growth is low, but more closely corresponding to how both economists and the public think of a recession.

Following Romer and Romer (2019), Otsu (2021) attempted to identify economic indicators of economic slack in Japan, which show peaks and troughs consistent with the reference dates. It used monthly labor-market indicators of composite indices of Japan. To measure slack in labor market, it used *deviation cycles* or *growth cycles*, that is, the departure from secular trends based on filtering methods. It used three types of filtering methods: the Christiano-Fitzgerald filter (Christiano and Fitzgerald, 2003), the Hamming-windowed filter (Iacobucci and Noullez, 2005) and the Butterworth filters (e.g. Gomez, 2001; Pollock, 2000). Canova (2007) gives a concise description of these methods.

Otsu (2021) found that 'Index of Non-Scheduled Worked Hours (Industries Covered)' is a promising variable to identify a recession in Japan. It also found that the unemployment rate, one of the useful indicators to identify the modern U.S. recessions in the literature, does not produce dates of the turning points consistent with the official reference dates of Japan.

To understand how the economic slack deepens, we need to quantify the degree of slack over time. The economic slack is measured by a *deviation cycle* as in Otsu (2021). One way to quantify the slacking degree is to compute probabilities of the economy in the economic slack over time, the so-called *smoothed probabilities*. Then, this paper uses Markov switching models with a deviation-cycle data in the labor market of Japan to compute such probabilities and examines how useful they are to understand the business cycle.

The main findings are as follows. First, a simple Markov switching model generates the smoothed probabilities that are closely related to the business cycle. Second, inclusion of lagged dependent variables as explanatory variables does not solve the model misspecification problem indicated by the conventional specification testing. Third, a Markov switching model fits the data with frequency components higher than seasonality better than those without them in terms of the specification testing. The models accepted by specification testing, however, do not generate the smoothed probabilities consistent with the business cycle.

The rest of the paper is organized as follows. In section 2, we briefly review simple Markov switching models used to be estimated. Section 3 discusses data for our analyses. We use 'Index of Non-Scheduled Worked Hours' which is one of the individual indicators of the composite indices, Japan, and found useful to identify a recession in Japan (see Otsu, 2021). In section 4, we examine the estimation results of the Markov switching models. We compute the smoothed probabilities and compare them with the officially published peaks and troughs of the business cycle to see whether the estimated probabilities are useful for recession identification. The final section is allocated to discussion.

2 Markov Switching Model

Following the recommendations in Hamilton (2016), we use the following simple Markov switching model. Suppose we have M different states. Let y_t an economic time-series variable at time t $(t=1, 2, \cdots, T)$ to be modeled:

$$\phi(L)y_t=\mu_i+\varepsilon_{it},\quad i=1, 2, \cdots, M, \tag{1}$$

$$\varepsilon_{it}\sim N(0,\ \sigma_i^2),\quad i=1, 2, \cdots, M, \tag{2}$$

$$\phi(L)=1-\sum_{j=1}^{K}\phi_{ij}L^j,\quad L^k y_t\equiv y_{t-k}, \tag{3}$$

where μ_i, σ_i and ϕ_{ij} are all unknown constant parameters.

Let the economic state at time t denoted by an unobserved variable, S_t. The transition between states is assumed to be governed by a first-order Markov process that is independent of ε_{it}. Then, the transition probability from i to j is:

$$Prob\,[S_t=j\,|\,S_{t-1}=i]=p_{ij},\quad i, j=1, 2, \cdots, M, \tag{4}$$

where

$$\sum_{j=1}^{M}p_{ij}=1. \tag{5}$$

Since the economic state is not observed, we need to infer it from past information. Let Ω_{t-1} the past information available at time t. Specifically, we set $\Omega_{t-1}=\{y_{t-1}, y_{t-2}, \cdots\}$. Then, we could calculate the probabilities of being in the state j, $S_t=j$, for $t=1, 2, \cdots, T$, by iteration as follows:

$$P[S_t=j\,|\,\Omega_{t-1}]=\sum_{i=1}^{M}Prob[S_t=j, S_{t-1}=i\,|\,\Omega_{t-1}] \tag{6}$$

$$=\sum_{i=1}^{M}Prob[S_t=j|S_{t-1}=i]P[S_{t-1}=i|\Omega_{t-1}]. \tag{7}$$

To find the joint density of y_t and S_t, we define the conditional density function of y_t for the state i as:

$$f(y_t\,|\,S_t=i, \Omega_{t-1})=\frac{1}{\sqrt{2\pi\sigma_i^2}}exp\left(-\frac{e_{it}^2}{2\sigma_i^2}\right), \tag{8}$$

where

$$e_{it}=\phi(L)y_t-\mu_{it},\quad i=1, 2, \cdots, M. \tag{9}$$

Then, the joint density of y_t and S_t can be written as:

$$f(y_t, S_t=i|\Omega_{t-1})=f(y_t|S_t=i, \Omega_{t-1})P[S_t=i|\Omega_{t-1}]. \tag{10}$$

We obtain the marginal density of y_t by summing over S_t:

$$f(y_t|\Omega_{t-1})=\sum_{j=1}^{M}f(y_t|S_t=j, \Omega_{t-1})P[S_t=j|\Omega_{t-1}]. \tag{11}$$

To compute eq.(11), we need stating values for $P[S_0|\Omega_0]$ in eq.(7). We can employ the following steady-state probabilities of S_t. Let the unconditional probabilities ρ_{it} of being in the state i and the transition matrix P^*. That is,

$$R_t=\begin{pmatrix}\rho_{1t}\\ \rho_{2t}\\ \vdots\\ \rho_{Mt}\end{pmatrix} \tag{12}$$

and

$$P^*=\begin{pmatrix}p_{11} & p_{21} & \cdots & p_{M1}\\ p_{12} & p_{22} & \cdots & p_{M2}\\ \vdots & \vdots & \ddots & \vdots\\ p_{1M} & p_{2M} & \cdots & p_{MM}\end{pmatrix}, \tag{13}$$

where $\sum_{i=1}^{M}\rho_{it}=1$ and $\sum_{j=1}^{M}p_{ij}=1$ for $i=1, 2, \cdots, M$. Then, a first-order Markov-chain property gives:

$$R_{t+1}=P^*R_t. \tag{14}$$

We obtain the steady-state probabilities by solving the eq.(14) under the conditions of $\sum_{i=1}^{M}\rho_{it}=1$ and $R_{t+1}=R_t$. Let the solutions $(\rho_1, \rho_2, \cdots, \rho_M)$. Then, we set:

$$P[S_0=i|\Omega_0]=\rho_i, \quad i=1, 2, \cdots, M. \tag{15}$$

Once we obtain y_t at the end of time t, we can update the $P[S_{t-1}=i|\Omega_{t-1}]$ in the following way:

$$P[S_t=i|\Omega_t]=\frac{f(y_t, S_t=i|\Omega_{t-1})}{f(y_t|\Omega_{t-1})} \tag{16}$$

$$=\frac{f(y_t|S_t=i, \Omega_{t-1})P[S_t=i|\Omega_{t-1}]}{\sum_{j=0}^{M} f(y_t|S_t=j, \Omega_{t-1})P[S_t=j|\Omega_{t-1}]}, \tag{17}$$

which is known as a *filtered probability* at time t in the literature. Combing this updating formula with eq.(7), we can iteratively compute the marginal density of y_t, eq.(11). Then, the log-likelihood function is given by

$$\mathcal{L}(\boldsymbol{\lambda})=\sum_{t=1}^{T} ln f(y_t|\Omega_{t-1}\ ;\ \boldsymbol{\lambda}), \tag{18}$$

where $\boldsymbol{\lambda}$ denotes a vector of the unknown constant parameters, $(\mu_i,\ \phi_{ik},\ p_{ij},\ \sigma_i)'$, $i=1, 2, \cdots, M$, $k=1, 2, \cdots, K$, and $j=1, 2, \cdots, M-1$. The unknown parameters can be estimated by maximizing the log-likelihood, eq.(18). In the later analysis, we set $M=2$. In this case, $\boldsymbol{\lambda}=(\mu_1, \mu_2, \phi_{1k}, \phi_{2k}, p_{11}, p_{22}, \sigma_1, \sigma_2)'$, $k=1, 2, \cdots, K$, and the steady-state probabilities in eq.(15) are given as follows:

$$\rho_1=\frac{1-p_{22}}{2-p_{11}-p_{22}}, \tag{19}$$

$$\rho_2=\frac{1-p_{11}}{2-p_{11}-p_{22}}. \tag{20}$$

To draw an inference about the economic states, we might use the filtered probabilities that are calculated with observed data along with estimates of the constant unknown parameters. It is possible, however, to obtain a better inference as more data accumulate. Suppose we have information at time $t+h$ to infer the economic states at time t. Consider the following joint probabilities:

$$Prob[S_t=j, S_{t+1}=i|\Omega_{t+h}]$$
$$=Prob[S_{t+1}=i|\Omega_{t+h}]Prob[S_t=j|S_{t+1}=i, \Omega_{t+h}]. \quad (21)$$

Let $\boldsymbol{Y}_{t+1}^{t+h}=\{y_{t+1}, y_{t+2}, \cdots, y_{t+h}\}$. Then, we have:

$$Prob[S_t=j|S_{t+1}=i, \Omega_{t+h}]$$
$$=Prob[S_t=j|S_{t+1}=i, \boldsymbol{Y}_{t+1}^{t+h}, \Omega_t] \quad (22)$$
$$=\frac{f(\boldsymbol{Y}_{t+1}^{t+h}, S_t=j|S_{t+1}=i, \Omega_t)}{f(\boldsymbol{Y}_{t+1}^{t+h}|S_{t+1}=i, \Omega_t)} \quad (23)$$
$$=\frac{f(\boldsymbol{Y}_{t+1}^{t+h}|S_{t+1}=i, S_t=j, \Omega_t)Prob[S_t=j|S_{t+1}=i, \Omega_t]}{f(\boldsymbol{Y}_{t+1}^{t+h}|S_{t+1}=i, \Omega_t)}.$$

Since S_{t+1} is inferred from S_t with exogenous probabilities in eq.(4), if S_t affects $\boldsymbol{Y}_{t+1}^{t+h}$ only through S_{t+1}, we have:

$$f(\boldsymbol{Y}_{t+1}^{t+h}|S_{t+1}=i, \Omega_t)=f(\boldsymbol{Y}_{t+1}^{t+h}|S_{t+1}=i, S_t=j, \Omega_t). \quad (24)$$

This gives a substantially simplified expression of eq.(23):

$$Prob[S_t=j|S_{t+1}=i, \Omega_{t+h}]=Prob[S_t=j|S_{t+1}=i, \Omega_t]. \quad (25)$$

Note that if S_t affects $y_{t+1}\in\boldsymbol{Y}_{t+1}^{t+h}$ through its state-dependent lags, that is, $\phi_{ij}\neq 0$ in eq. (3), then, eq. (24) holds only approximately, and eq. (25) involves an approximation (see Kim, 1994, pp. 9-10, for discussion). If eq.(24) holds either exactly or approximately good enough, eq.(21) can be written as:

$$Prob[S_t=j, S_{t+1}=i|\Omega_{t+h}]$$
$$=\frac{Prob[S_{t+1}=i|\Omega_{t+h}]Prob[S_t=j, S_{t+1}=i|\Omega_t]}{Prob[S_{t+1}=i|\Omega_t]} \quad (26)$$
$$=\frac{Prob[S_{t+1}=i|\Omega_{t+h}]Prob[S_{t+1}=i|S_t=j]P[S_t=j|\Omega_t]}{Prob[S_{t+1}=i|\Omega_t]}.$$

Then, we can infer S_t from data observed through $t+h$:

$$Prob[S_t=j|\Omega_{t+h}]=\sum_{i=1}^{M}Prob[S_t=j,\, S_{t+1}=i|\Omega_{t+h}]. \tag{27}$$

This is known as an h-time-ahead smoothed inference. The algorithm in eq.(26) is developed by Kim (1994) (also see Ch.4 in Kim and Nelson, 1999). Given $Prob[S_{t+h}|\Omega_{t+h}]$, the iteration along with eq.(26) and eq.(27) gives rise to the h-time-ahead smoothed probabilities, $Prob[S_t|\Omega_{t+h}]$, $t=t+h-1$, $t+h-2$, $\cdots$, 1. When we extend $t+h$ to T, we obtain the *full-sample smoother*, proposed by Hamilton (1989), which we use in the later analysis. That is,

$$Prob[S_t=j|\Omega_T]=\sum_{i=1}^{M}Prob[S_t=j,\, S_{t+1}=i|\Omega_T]. \tag{28}$$

Finally, we consider the following specification of eq.(4):

$$Prob[S_t=j|S_{t-1}=i]=\boldsymbol{\Phi}(\alpha_{ij}), \qquad i,\, j=1,\, 2,\, \cdots,\, M, \tag{29}$$

where $\boldsymbol{\Phi}$ is the standard normal cumulative distribution, and α_{ij} is a constant parameter. In case of two states, $M=2$, we only need to estimate p_{11} and p_{22} in eq.(4). Thus,

$$Prob[S_t=i|S_{t-1}=i]=\boldsymbol{\Phi}(\alpha_i), \qquad i=1,\, 2. \tag{30}$$

3 Data

We use the same data as in Otsu (2021): 'Index of Non-Scheduled Worked Hours' (NSWH) that is included in the individual indicators of coincident composite indices, complied by Economic and Social Research Institute (ESRI) affiliated with the Cabinet Office, Government of Japan. ESRI routinely examines and revises the composition of indicators. The latest revision, the 13th revision, was made in March 2021.

Otsu (2021) used NSWH to estimate economic slack for the period of January

1980 to January 2020. As Romer and Romer (2019, see p. 12) discussed, a recession can be defined as a sustained decline in the rate of growth of aggregate economic activity relative to its long-term trend. Therefore, the ‘slack’ concept is better suited to the *growth* cycles than the *level* cycles. In this interpretation, the growth cycles are deemed detrended business cycles because a recession is interpreted as a part of business cycle. Otsu (2021) used bandpass filters to suppress a secular trend and noise components and to extract detrended growth cycles, that is, the deviation cycles. Following Burns and Mitchell (1946), the business cycles are assumed to range from 18 months (1.5 years) to 96 months (8 years), while the secular trend corresponds to the longer-cycle components and the noise to the shorter ones. The details are given in Otsu (2021).

A caveat is in order. In January 2018, it was revealed that officials at Ministry of Health, Labor and Welfare had incorrectly conducted fundamental statistical survey on labor-related conditions since 2004. Then, our data set is susceptible to this incorrect compilation. According to Economic and Social Research Institute (ESRI), affiliated with the Cabinet Office, Government of Japan, it has used the corrected values published by the Monthly Labor Survey since January 2019 for the time period from January 2012 onward as a remedy for the faulty data problem. The earlier part of the series than the correction is connected by a link coefficient method. Such a remedy makes these series good enough for our analysis.

To find how useful the estimated probabilities of economic states are to understand the business cycle, we compare the probabilities with the official reference dates of Japan. Table 1 shows the reference dates of peaks and troughs identified by ESRI. It also contains periods of expansion, contraction, and duration of a complete cycle (trough to trough). There are 15 peak-to-trough phases identified after World War II. In these phases, the average period is about 36 months for expansion, 16 for contraction, and 52 for the complete cycle. The reference dates are available on the website of ESRI[1].

Table 2 gives a summary of descriptive statistics of dependent variables used in the later analyses. The sample period ranges from February 1980 to January 2020, and the total number of observation is 480. In the second and the third column, we compute the statistics with the rate of change of the level data: the original official data and the data with the components higher than the seasonal frequency being filtered out by a Butterworth filter as in Otsu (2020), respectively. Both data show the same mean values, but the standard deviation is smaller for the latter by 43%: the frequencies higher than seasonality give rise to a large variation. The skewness and the kurtosis indicate that they are away from a normal distribution. The Jarque-Bera statistic suggests non-normality.

Since the deviation-cycle data could take zero values, we use the first difference instead of the rate of change. The fourth column indicates the mean of the business-cycle components is zero as expected. This is because the components are computed as a deviation from a trend in the filtering process. Therefore, even if we add the higher frequencies to the business cycle, the mean would be zero, which is the case as shown in the fifth column. Comparing the standard deviations in the fourth and the fifth columns, we find that the higher frequencies contribute to a large variation, as pointed out above. Although the skewness and the kurtosis in the fourth column are smaller in absolute value than those in the fifth, they still indicate that the shape of the sample distribution is different from that of a normal curve. In fact, the Jarque-Bera testing rejects the normality.

1） Indexes of Business Conditions: https://www.esri.cao.go.jp/en/stat/di/di-e.html, Dec. 2, 2021.

4 Empirical Analysis

4.1 Preliminary Results

We begin with the estimation results based on the original series of 'Index of Non-Scheduled Worked Hours' for the period from January 1980 to January 2020. The dependent variable is the rate of change. Table 3 shows Maxmimum Likelihood estimates of the model when $K=0$ in section 2. The conventional wisdom says that estimates of α_1 and σ_1 are statistically significant, while those of other parameters are either insignificant or marginally significant. Although the estimates of μ_1 and μ_2 are not statistically significant, the positive estimate of μ_1 suggests the increase (0.12% per month) in the non-scheduled worked hours in the state one, while the negative one (-1.10% per month) of μ_2 indicates the decrease in the state two.

Table 3 also shows the specification testing statistics studied by White (1987) and Hamilton (1996). The testing does not reject the null hypothesis of the first-order autoregressive (AR(1)) effect in residuals of the state two. The first-order autoregressive conditional heteroskedasticity (ARCH(1)) in residuals is marginally significant in both states. The joint effects of AR(1) and ARCH(1) across states emerge. These indicate the possibility of misspecification of the econometric model.

Table 4 and Table 5 show the estimation results when $K=1$ and $K=2$. The estimates of the constant terms, μ_1 and μ_2, are either not statistically significant or significant only marginally. The estimates of the autoregressive coefficients, ϕ_{ij}, i, $j=1$, 2, are significant in the state two, but marginal in the state one. Since the average of the dependent variable is -0.025%, the state one when $K=1$ indicates the increase of 0.151% on average and the state two the decrease of -0.219%. When $K=2$, the non-scheduled worked hours increase by 0.160% per month in the state one and decreases by -0.096% in the state two. When $K=0$ and $K=1$, the worked hours change more rapidly when decrease than when increase. On the contrary, the average decreasing rate in the state two is, in absolute value, smaller than the average increasing rate in the state one.

In Table 4, the specification testing shows no AR(1) effects in residuals, but marginal statistical significance in ARCH(1) and the joint effects. When $K=2$ in Table 5, it indicates no AR(1) in the state two and no ARCH(1) in the state one, but marginal significance of AR(1) effect in the state one and of ARCH(1) in the state two. The joint testing indicates the presence of AR(1) and ARCH(1) effects. The statistical testing gives mixed results on the misspecification. It seems, however, that the model with $K=1$ is comparatively better specified among the three cases.

To examine effects of high-frequency components on estimates, we separate the series into components with lower than 13 months per cycle and those with higher than 12 months per cycle, that is, seasonality. We use a bandpass filter, the tangent-based Butterworth filter, to extract a designated frequency band. In Table 6, we suppress the components that have frequencies higher than seasonality. Since the NSWH, officially published, is a seasonally adjusted series, the suppressed components are supposed to consist of all frequencies less than one year. Then, the specification testing tells that the model is misspecified. In Table 7, we only use the higher frequency components to estimate the model. Then, AR(1) effect is found in the state one, and the joint testing of AR(1) and ARCH(1) indicates marginal statistical significance. But, other statistics show no misspecification. Therefore, the higher-frequency components make the specification testing less likely to reject the null hypothesis of no misspecification.

Figure 1 shows the smoothed probabilities of the increasing NSWH index, based on estimates in Table 3 through Table 6. The shaded areas in the figure indicate the periods from peaks to troughs of the economy. Thus, we expect to observe that the probability of the increasing index is high at the peak, that is, the start of an area, and getting lower and lower for some period toward the trough, may it not be monotonically. The first panel shows the smoothed probabilities when $K=0$. They well correspond to the movement of the index. In addition, it indicates that the economy is likely to be in the state of decreasing NSWH, during the recession periods of 1991 to 1993 and 2008 to 2009. Note that the smoothed probabilities of eq. (25) in section 2 do not involve any computational approximation in this case. In the second panel of $K=1$, the smoothed probabilities give a similar result, but a better description for the periods of 1997 to 1999 and 2000 to 2002 because they clearly indicate the economy is in the state two, the decreasing NSWH.

In the third panel, the smoothed probabilities fluctuate a lot due to smaller transition probabilities: $p_{11}=0.8404$ and $p_{22}=0.8303$. It is hard to judge whether they correspond to the business cycle. In the fourth panel, we suppress the frequency components higher than seasonality. In the state one, the NSHW index changes on average by -0.031 points when the mean of the dependent variables is -0.020, while in the state two by 0.003. Thus, the state two would indicate the increasing state, and its smoothed probabilities appear in the fourth panel. They move quite differently from those in other panels. It is hard to understand their relation with the business cycle or the index because the economy is likely to be in the state two (increasing worked hours) during the recession after 2000.

4.2 Deviation Cycles

In Table 8, we apply the Markov switching model to the business-cycle components, which are extracted with a Butterworth filter as in Otsu (2021), the parameter estimates are statistically significant at 5% level. The specification testing indicates possibility of model misspecification. Table 9 shows the estimation result when the model have a lagged dependent variable as an explanatory variable. The estimates are statistically significant, except those of the constant terms. But, the specification testing rejects the null hypotheses of no AR(1) and no ARCH(1) effects: there is no improvement in model specification.

When the higher frequency components are included, as in the previous section 4.1, the model specification somewhat improves. The specification testing in Table 10 accepts no AR(1) effect in state one, and no ARCH(1) effect in state two. But, it still indicates misspecification possibilities. In Table 11, even when a lagged dependent variable is added to the model, the misspecification remains. Further, many of the parameter estimates are statistically not significant.

Since the estimate on ϕ_{11} is not significant, we estimate the model with a constraint of $\phi_{11}=0$ in Table 12. Then, the specification testing only indicates ARCH(1) effect in state one, and marginally rejects the joint hypotheses of no AR(1) and no ARCH(1) effects. In experiments, we used more lagged dependent variables as explanatory variables, but we only obtained unreliable estimates and even no normal convergence.

Figure 2 shows the smoothed probabilities computed with estimates in Tables 8 - 10 and Table 12. In the first panel, the smoothed probabilities are very high, almost 1, at the peaks. Although they are also high at the troughs, they sharply go down once and then go up toward the troughs. Therefore, it is possible to interpret that the smoothed probabilities predict the end of recessions. The graph in the first panel is comparable with that of the second panel in Figure 1, but gives even better description of the 1980s. The second panel shows that the smoothed probabilities move almost oppositely to those of the first panel. Thus, the probabilities seem lag behind the business cycle except the period of the 1990s. Note that the graphs in the first and the second panels are drawn based on the estimates of the models of which the conventional specification testing indicates misspecification.

The smoothed probabilities in the third and the fourth panels only capture the recession from February 2008 to March 2009. In other periods, the smoothed probabilities indicate that the economy is in the state of rising non-scheduled worked hours. That is, they have no relation with the business cycle. Although the specification testing tells that the models are less likely to be misspecified, particularly in Table 12, the generated smoothed probabilities seem not useful for the business-cycle analyses.

In sum, when we estimate a simple Markov switching model estimated with the filtered data that have only the business-cycle components, the specification testing rejects the null hypotheses of no misspecification. But the estimates generates the smoothed probabilities which are closely related to the business cycle. Further, inclusion of higher frequency components make the specification testing less likely to indicate misspecification of the econometric models that produce the smoothed probabilities not easily understood in terms of the business cycle.

5 Discussion

This paper attempts to estimate the smoothed probabilities of economic slack in the labor market of Japan and examines how useful they are to understand the business cycle. We use Markov switching models to estimate them with a deviation-cycle data. The main findings are as follows. First, a simple Markov switching model generates the smoothed probabilities that are closely related to the business cycle. Second, inclusion of lagged dependent variables as explanatory variables does not solve the model misspecification problem indicated by the conventional specification testing. Third, a Markov switching model fits the data with frequency components higher than seasonality better than those without them in terms of the specification testing. The models accepted by specification testing, however, do not generate the smoothed probabilities consistent with the business cycle.

Two final remarks are in order. First, our results suggest that statistical and economic criteria might contradict each other. In another word, a good model in a statistical sense does not necessarily generate meaningful implication in terms of economics.

As the American Statistical Association (ASA) noted in the statement on statistical significance and *p*-values (Wasserstein and Lazar, 2016), we should not drive final conclusions solely from results of the null-hypothesis significance testing (NHST). That is, *p*-values above or below 0.05 neither prove nor disprove of the reality of anything. Therefore, we need further investigations, without any preconceived belief, on the usefulness of the estimated smoothed probabilities in terms of economics: the most pernicious effect of NHST is the delusive belief that statistical significance constitutes proof, as pointed out by Matthews (2021).

Secondly, it is known that statistical conditions, which the standard asymptotic distributional theory assumes, are not satisfied in statistical testing of Markov switching models, particularly on numbers of states: the models are plagued with unidentified nuisance parameters and identically zero scores under the null hypothesis. Thus, nonstandard tests are developed in the literature that includes Hansen (1992), Garcia (1998), Cho and White (2007) and Qu and Zhuo (2021). Then, it is interesting to see whether such a nonstandard testing leads to finding statistical models that produce the smoothed probabilities allowing sensible economic interpretations.

Finally, although we find a simple Markov switching model produces smoothed probabilities interpretable in terms of the business cycle, we should make sure that such a simple econometric model works well for other business-cycle indices or indicators. These are left for the ongoing research.

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Table 1 Reference Dates of Business Cycles: Japan

Date (month, year)				Number of Periods (in months)		
Peak		Trough		Expansion	Contraction	Duration
June,	1951	October,	1951	—	4	—
January,	1954	November,	1954	27	10	37
June,	1957	June,	1958	31	12	43
December,	1961	October,	1962	42	10	52
October,	1964	October,	1965	24	12	36
July,	1970	December,	1971	57	17	74
November,	1973	March,	1975	23	16	39
January,	1977	October,	1977	22	9	31
February,	1980	February,	1983	28	36	64
June,	1985	November,	1986	28	17	45
February,	1991	October,	1993	51	32	83
May,	1997	January,	1999	43	20	63
November,	2000	January,	2002	22	14	36
February,	2008	March,	2009	73	13	86
March,	2012	November,	2012	36	8	44
October,	2018*	May,	2020*	71	—	—

* Provisional date, as of July 30, 2020.
Source: *Indexes of Business Conditions*, Economic and Social Research Institute, Cabinet Office, Government of Japan, November 30, 2021.

Table 2 Descriptive Statistics of Dependent Variables

Statistic	Rate of Change (%)		First Difference	
	Official Data (S.A.)*	Passed Band $[0, 2\pi/13]$	Passed Band $[2\pi/96, 2\pi/18]$	Passed Band $[2\pi/96, 2\pi/2]$
mean	-0.0202	-0.0202	0.0000	0.0000
(standard error)	(0.0556)	(0.0318)	(0.0440)	(0.0525)
standard deviation	1.2185	0.6962	0.9650	1.1495
(standard error)	(0.1593)	(0.0627)	(0.0728)	(0.1235)
skewness	-0.7454	-1.3371	0.0017	-0.4994
(standard error)	(0.1111)	(0.1111)	(0.1111)	(0.1111)
kurtosis	6.5430	9.0725	3.9447	5.2054
(standard error)	(0.2201)	(0.2201)	(0.2201)	(0.2201)
maximum value	3.2508	2.3093	3.4209	3.4941
minimum value	-7.1763	-3.7469	-3.3436	-5.7733
Jarque-Bera Stat.**	295.5017	880.5278	17.8505	117.2285
(*p*-value)	(0.0000)	(0.0000)	(0.0001)	(0.0000)
Sample Period: Feb. 1980 to Jan. 2020			# of obs.: 480	

Note: * Seasonally Adjusted series. ** See Jarque and Bera (1987).

Table 3　Estimation of Markov Switching Model: lag $K=0$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	*p*-value
μ_1	0.118955	0.175755	0.677	0.4985
μ_2	-1.096310	1.267497	-0.865	0.3871
α_1	2.214343	0.425981	5.198	0.0000
α_2	1.307382	0.822758	1.589	0.1121
σ_1	1.009709	0.137952	7.319	0.0000
σ_2	1.936229	1.071101	1.808	0.0707
Log-Likelihood value: -739.8542				# of obs.: 480
Implied transition probability of staying		in state one: 0.9866;	in state two: 0.9045	

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	*p*-value
No AR(1) in residuals	in state one	0.7468	1	0.3875
	in state two	13.9268	1	0.0002
No ARCH(1) in residuals	in state one	5.1498	1	0.0232
	in state two	6.4100	1	0.0113
No AR(1) and ARCH(1) in both states		20.2367	4	0.0004

Note: The dependent variable is the rate of change.
　* Heteroskedastic consistent estimates.　** See White (1987) and Hamilton (1996).

Table 4　Estimation of Markov Switching Model: lag $K=1$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	*p*-value
μ_1	0.145106	0.073330	1.979	0.0478
ϕ_{11}	-0.235354	0.123192	-1.910	0.0561
μ_2	-0.206045	0.207374	-0.994	0.3204
ϕ_{21}	0.533912	0.186637	2.861	0.0042
α_1	1.914620	0.323686	5.915	0.0000
α_2	1.397630	0.298379	4.684	0.0000
σ_1	0.938806	0.062638	14.988	0.0000
σ_2	1.431189	0.203039	7.049	0.0000
Log-Likelihood value: -726.5472				# of obs.: 479
Implied transition probability of staying		in state one: 0.9722;	in state two: 0.9189	

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	*p*-value
No AR(1) in residuals	in state one	1.2557	1	0.2625
	in state two	1.2306	1	0.2673
No ARCH(1) in residuals	in state one	4.7044	1	0.0301
	in state two	6.5763	1	0.0103
No AR(1) and ARCH(1) in both states		11.8272	4	0.0187

Note: The dependent variable is the rate of change.
　* Heteroskedastic consistent estimates.　** See White (1987) and Hamilton (1996).

Table 5 Estimation of Markov Switching Model: lag $K=2$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	p-value
μ_1	0.143496	0.121809	1.178	0.2388
ϕ_{11}	-0.445443	0.166390	-2.677	0.0074
ϕ_{12}	-0.228365	0.122723	-1.861	0.0628
μ_2	-0.076363	0.082629	-0.924	0.3554
ϕ_{21}	0.460431	0.131414	3.504	0.0005
ϕ_{22}	0.326100	0.084314	3.868	0.0001
α_1	0.996271	0.863142	1.154	0.2484
α_2	0.955174	0.322274	2.964	0.0030
σ_1	0.963335	0.071872	13.404	0.0000
σ_2	0.994711	0.159658	6.230	0.0000
Log-Likelihood value: -720.9461				# of obs.: 478
Implied transition probability of staying	in state one: 0.8404 ;	in state two: 0.8303		

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	p-value
No AR(1) in residuals	in state one	4.0309	1	0.0447
	in state two	0.4196	1	0.5171
No ARCH(1) in residuals	in state one	2.2403	1	0.1345
	in state two	6.5791	1	0.0103
No AR(1) and ARCH(1) in both states		16.5823	4	0.0023

Note: The dependent variable is the rate of change.
 * Heteroskedastic consistent estimates. ** See White (1987) and Hamilton (1996).

Table 6 Estimation of Markov Switching Model: lag $K=1$, passband $=\left[0, \frac{2\pi}{13}\right]$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	p-value
μ_1	-0.011628	0.005579	-2.084	0.0371
ϕ_{11}	0.941869	0.012344	76.302	0.0000
μ_2	0.022683	0.025617	0.885	0.3759
ϕ_{21}	0.989060	0.034801	28.420	0.0000
α_1	1.828202	0.108519	16.847	0.0000
α_2	1.418384	0.185042	7.665	0.0000
σ_1	0.066488	0.004929	13.490	0.0000
σ_2	0.283458	0.036483	7.770	0.0000
Log-Likelihood value : 361.8797				# of obs.: 479
Implied transition probability of staying	in state one : 0.9662 ;	in state two: 0.9220		

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	p-value
No AR(1) in residuals	in state one	288.86232	1	0.0000
	in state two	177.31453	1	0.0000
No ARCH(1) in residuals	in state one	130.00234	1	0.0000
	in state two	39.72080	1	0.0000
No AR(1) and ARCH(1) in both states		427.82944	4	0.0000

Note: The dependent variable is the rate of change.
 * Heteroskedastic consistent estimates. ** See White (1987) and Hamilton (1996).

Table 7　Estimation of Markov Switching Model: $\text{lag}\,K=1$, passband $=\left[\frac{2\pi}{12}, \frac{2\pi}{2}\right]$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	*p*-value
μ_1	-0.005808	0.036867	-0.158	0.8748
ϕ_{11}	-0.348756	0.055134	-6.326	0.0000
μ_2	-0.006848	0.150284	-0.046	0.9637
ϕ_{21}	-0.096412	0.169189	-0.570	0.5688
α_1	1.614109	0.423039	3.816	0.0001
α_2	0.730825	0.256415	2.850	0.0044
σ_1	0.720943	0.071872	10.031	0.0000
σ_2	1.532536	0.193351	7.926	0.0000
Log-Likelihood value: -621.9115			# of obs.: 479	
Implied transition probability of staying		in state one : 0.9467;	in state two: 0.7676	

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	*p*-value
No AR(1) in residuals	in state one	9.9882	1	0.0016
	in state two	0.0092	1	0.9234
No ARCH(1) in residuals	in state one	2.1377	1	0.1437
	in state two	1.4615	1	0.2267
No AR(1) and ARCH(1) in both states		12.6773	4	0.0130

Note: The dependent variable is the first difference.
　* Heteroskedastic consistent estimates.　** See White (1987) and Hamilton (1996).

Table 8　Estimation of Markov Switching Model: $\text{lag}\,K=0$, passband $=\left[\frac{2\pi}{96}, \frac{2\pi}{18}\right]$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	*p*-value
μ_1	0.093496	0.021771	4.295	0.0000
μ_2	-0.289540	0.117931	-2.455	0.0141
α_1	2.031778	0.105717	19.219	0.0000
α_2	1.487239	0.108531	13.703	0.0000
σ_1	0.284500	0.011534	24.666	0.0000
σ_2	1.035998	0.073706	14.056	0.0000
Log-Likelihood value: -266.6184			# of obs.: 480	
Implied transition probability of staying		in state one: 0.9789;	in state two: 0.9315	

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	*p*-value
No AR(1) in residuals	in state one	329.2961	1	0.0000
	in state two	140.6161	1	0.0000
No ARCH(1) in residuals	in state one	135.8444	1	0.0000
	in state two	60.3336	1	0.0000
No AR(1) and ARCH(1) in both states		446.1956	4	0.0000

Note: The dependent variable is the first difference.
　* Heteroskedastic consistent estimates.　** See White (1987) and Hamilton (1996).

Table 9 Estimation of Markov Switching Model: lag $K=1$, passband $=\left[\frac{2\pi}{96}, \frac{2\pi}{18}\right]$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	p-value
μ_1	-0.009204	0.006683	-1.377	0.1684
ϕ_{11}	0.947211	0.038573	24.557	0.0000
μ_2	0.028452	0.037744	0.754	0.4510
ϕ_{21}	0.997802	0.034484	28.935	0.0000
α_1	1.884400	0.318420	5.918	0.0000
α_2	1.272363	0.127120	10.009	0.0000
σ_1	0.061765	0.012973	4.761	0.0000
σ_2	0.238799	0.058994	4.048	0.0001
Log-Likelihood value: 461.3215				# of obs.: 479
Implied transition probability of staying		in state one: 0.9702;	in state two: 0.8984	

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	p-value
No AR(1) in residuals	in state one	326.0702	1	0.0000
	in state two	166.5462	1	0.0000
No ARCH(1) in residuals	in state one	161.5971	1	0.0000
	in state two	61.3015	1	0.0000
No AR(1) and ARCH(1) in both states		440.6322	4	0.0000

Note: The dependent variable is the first difference.
* Heteroskedastic consistent estimates. ** See White (1987) and Hamilton (1996).

Table 10 Estimation of Markov Switching Model: lag $K=0$, passband $=\left[\frac{2\pi}{96}, \frac{2\pi}{2}\right]$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	p-value
μ_1	0.039724	0.049567	0.050	0.4229
μ_2	-4.638722	0.429257	-10.806	0.0000
α_1	2.843678	0.300776	9.454	0.0000
α_2	0.632870	0.674055	0.939	0.3478
σ_1	1.066225	0.039122	27.254	0.0000
σ_2	0.912002	0.192502	4.738	0.0000
Log-Likelihood value: -720.6579				# of obs.: 480
Implied transition probability of staying		in state one: 0.9978;	in state two: 0.7366	

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	p-value
No AR(1) in residuals	in state one	0.0761	1	0.7827
	in state two	12.7162	1	0.0004
No ARCH(1) in residuals	in state one	7.2408	1	0.0071
	in state two	0.3163	1	0.5738
No AR(1) and ARCH(1) in both states		283.6656	4	0.0000

Note: The dependent variable is the first difference.
* Heteroskedastic consistent estimates. ** See White (1987) and Hamilton (1996).

Table 11　Estimation of Markov Switching Model: lag $K=1$, passband $=\left[\frac{2\pi}{96}, \frac{2\pi}{2}\right]$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	*p*-value
μ_1	0.037448	0.100012	0.374	0.7081
ϕ_{11}	-0.104763	0.182284	-0.575	0.5655
μ_2	-0.123096	0.249174	-0.494	0.6213
ϕ_{21}	0.552941	0.220038	2.513	0.0120
α_1	2.164467	1.037087	2.087	0.0369
α_2	1.204721	0.446357	2.699	0.0070
σ_1	0.984642	0.107822	9.132	0.0000
σ_2	1.614181	0.631742	2.555	0.0106
Log-Likelihood value: -714.0702				# of obs.: 479
Implied transition probability of staying		in state one: 0.9848;	in state two: 0.8858	

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	*p*-value
No AR(1) in residuals	in state one	6.6071	1	0.0102
	in state two	3.1374	1	0.0765
No ARCH(1) in residuals	in state one	7.0750	1	0.0078
	in state two	0.2480	1	0.6185
No AR(1) and ARCH(1) in both states		13.6935	4	0.0083

Note: The dependent variable is the first difference.
 * Heteroskedastic consistent estimates.　** See White (1987) and Hamilton (1996).

Table 12　Estimation of Markov Switching Model: $K=1$, $\phi_{11}=0$, passband $=\left[\frac{2\pi}{96}, \frac{2\pi}{2}\right]$

Parameter	Estimate (1)	Standard Error (2)*	(1)/(2)	*p*-value
μ_1	0.015548	0.056555	0.275	0.7834
μ_2	-0.087890	0.328173	-0.268	0.7888
ϕ_{21}	0.598596	0.248715	2.407	0.0161
α_1	2.605718	0.557654	4.673	0.0000
α_2	1.549390	0.851682	1.819	0.0689
σ_1	1.021331	0.053686	19.024	0.0000
σ_2	1.750022	0.202081	8.660	0.0000
Log-Likelihood value: -714.0702				# of obs.: 479
Implied transition probability of staying		in state one: 0.9954;	in state two: 0.9394	

Specification Testing**

Null Hypothesis		χ^2 stat.	d.f.	*p*-value
No AR(1) in residuals	in state one	1.7539	1	0.1854
	in state two	1.9011	1	0.1680
No ARCH(1) in residuals	in state one	9.0109	1	0.0027
	in state two	0.7520	1	0.3858
No AR(1) and ARCH(1) in both states		11.1123	4	0.0253

Note: The dependent variable is the first difference.
 * Heteroskedastic consistent estimates.　** See White (1987) and Hamilton (1996).

Figure 1 Smoothed Probabilities: Non-Scheduled Worked Hours (Seasonally Adjusted)

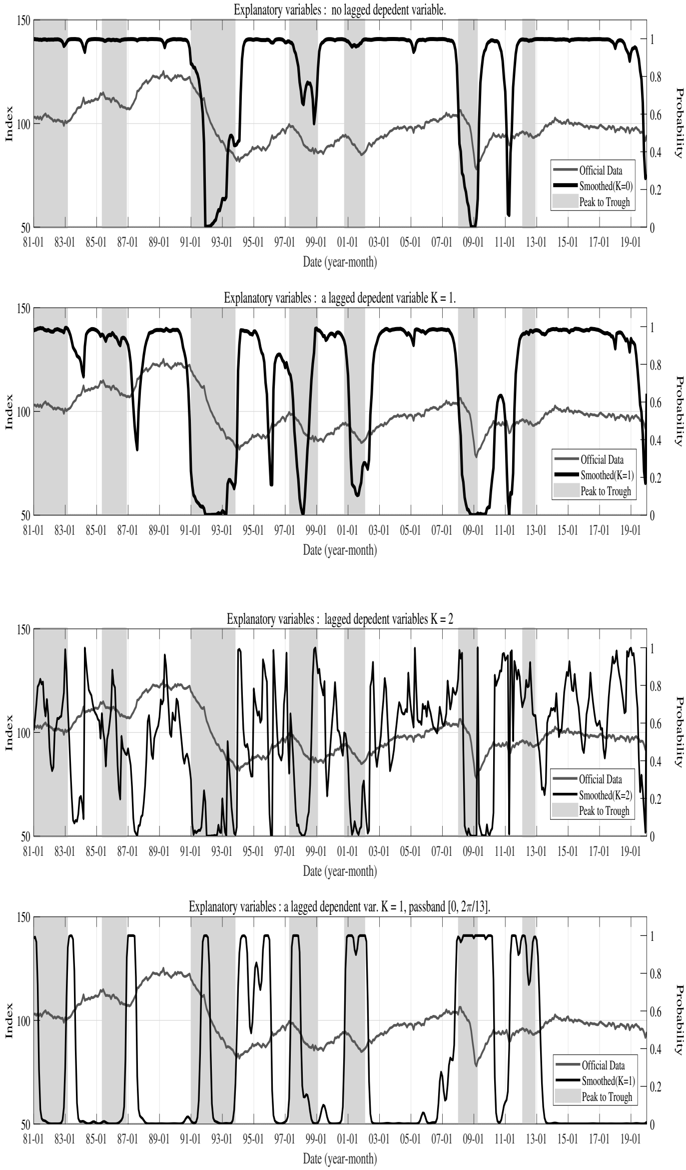

Figure 2　Smoothed Probabilities: Non-Scheduled Worked Hours (Business Cycle)